## Exercise 159

For the following exercises, solve the trigonometric equations on the interval $0 \leq \theta<2 \pi$.

$$
\sqrt{3} \cot \theta+1=0
$$

## Solution

$$
\begin{aligned}
& \sqrt{3} \cot \theta+1=0 \\
& \sqrt{3} \cot \theta=-1 \\
& \cot \theta=-\frac{1}{\sqrt{3}} \\
& \frac{1}{\tan \theta}=-\frac{1}{\sqrt{3}} \\
& \tan \theta=-\sqrt{3}
\end{aligned}
$$

The tangent of an angle is negative in quadrants II and IV.


Taking the inverse tangent of $-\sqrt{3}$ gives $-60^{\circ}$, or $-\pi / 3$ radians. This is $\alpha$ in the figure.

$$
\alpha=-\frac{\pi}{3}
$$

To obtain the angle to the point in the second quadrant, add $\pi$ to $\alpha$.

$$
\pi+\alpha=\frac{2 \pi}{3}
$$

Since every angle has to be between 0 and $2 \pi$, use $\alpha+2 \pi=5 \pi / 3$ rather than $\alpha$. Therefore,

$$
\theta=\left\{\frac{2 \pi}{3}, \frac{5 \pi}{3}\right\}
$$

